# LAYING OUT A TWO-CUT MITER

by Mike Bryant

Before we go any further, remember this technique only applies to parallel-sided wood pipes—Bourdons, Tibias, and the like.

In order to achieve a 90° miter with two cuts, each cut must be 22½°. There are several ways to lay out that angle, but the simplest and most repeatable method (for accuracy) will be to make a template with a perfect 22½° leg and a fence to fit up against the face of the pipe.

There are also several ways to get the angle correct for the template, so we've put instructions in the Appendix at the end of this article that cover just about any means you might have to establish an accurate angle and cut a template (power miter saw, table saw, jig saw, circular saw, etc.).

First we want to talk about locating the cuts on the pipe. Everything we discussed in the original article holds true, but because we have two cuts to make, things are a bit more complex. You will still have the limiting factor of total allowable pipe height, but unlike the single-cut method, where the top of the cut (on the back of the pipe) sets the height of the finished pipe, that isn't so with a two-cut miter. There are a couple more steps involved.

Before we go on, the pipe used for illustration in the pictures was an orphan that was badly damaged and beyond repair long before it ever came into our hands. We didn't spoil a perfectly good pipe just to illustrate this article.

For reference, we'll refer to the portion of the pipe that will remain vertical as the pipe, the section that will wind up horizontal as the mitered section, and the portion that is cut out to make the miter as the wedge.

After we reassemble the pipe, we want the lower surface of the mitered section to wind up aligned with the highest point of the pipe and the front of the pipe to align with the mitered section cut in the same way (see the next picture).

Figure 1 - The bottom of the mitered section aligns with the top of the cut at the back of the pipe. That's why the dimensions of the wedge are important. If the wedge was either larger or smaller, the alignment (and visual effect) would be off.

How do we know where to make the cuts? The wedges will vary in size; the larger the pipe, the larger the wedge. If we were to use the same dimensions for the wedge on all pipes, the proportions would wind up way out of balance. To maintain correct proportions, every pipe will require individual measurement and calculation for the locations of the cuts, and the dimensions of the wedge.

With the single-cut method, we only had to concern ourselves with the location of the top of the cut. In this case, though, we need to take into account the depth of the pipe as well. As the edge of the level shows in the picture above, the top of the lower cut is even with the bottom of the mitered section. In order to have all the tops level, you need to include the depth of the pipe in your calculation.

After you've calculated the maximum pipe height as we described in the original article, you subtract the depth of the pipe to determine the cut point. For example, if the maximum allowable height is 81" above the top surface of the chest, and the depth of the pipe is 6", the top of the lower cut can be no higher than 75" up the pipe.

Once you locate the point for the top of the lower cut, mark it at the back of the pipe.

If you're mitering multiple pipes and want the tops to remain level, use the smallest pipe as the base for your calculation, and step the lower cut down as the depth of the pipe increases. The amount you'll step down for each pipe will be the difference in depth between the two pipes. If the next larger pipe is 6¼" deep, the target point for that pipe will be 74¾" up, and so on.

It's easy to figure out where the first cut should go, but what about the second (upper) cut? It's time to revisit high-school geometry. Let's list out what we know; that will lead us to what we need to find out.

We know:

- the depth of the pipe, front to back;
- the finished angle of the mitered section to the pipe must be 90°;
- the cuts must align as described above;
- to achieve a 90° turn, we can use any combination of equal cuts whose angles add up to 45° (we will use two cuts of 22½° each).

We need to find out:

- The top of the lower cut (we've already determined how to do that—now we just need to know where);
- The bottom of the lower cut;
- The location of the upper cut; and
- We need a method of validating our measurements before we make the first cut.

Imagine if you were to cut the pipe straight across then lay out the two pieces at 90° to each other with the inside corners touching, you could then draw a line connecting the two outside corners. That's the dimension we need for the distance between the ends of the "long" side of the wedge that forms the miter. It is also apparent that it is the hypotenuse of a right triangle whose other two legs are equal to the depth of the pipe. Enter Pythagoras and his theorem for right triangles:

$$a^2 + b^2 = c^2$$

…where *a* and *b* are the lengths of the sides and *c* is the hypotenuse. If the pipe is 8" deep, then:

$$8^2 + 8^2 = c^2$$
$$64 + 64 = c^2$$
$$128 = c^2$$
$$c = \sqrt{128} = 11.314\text{"} \;\; (11\tfrac{5}{16}\text{"})$$

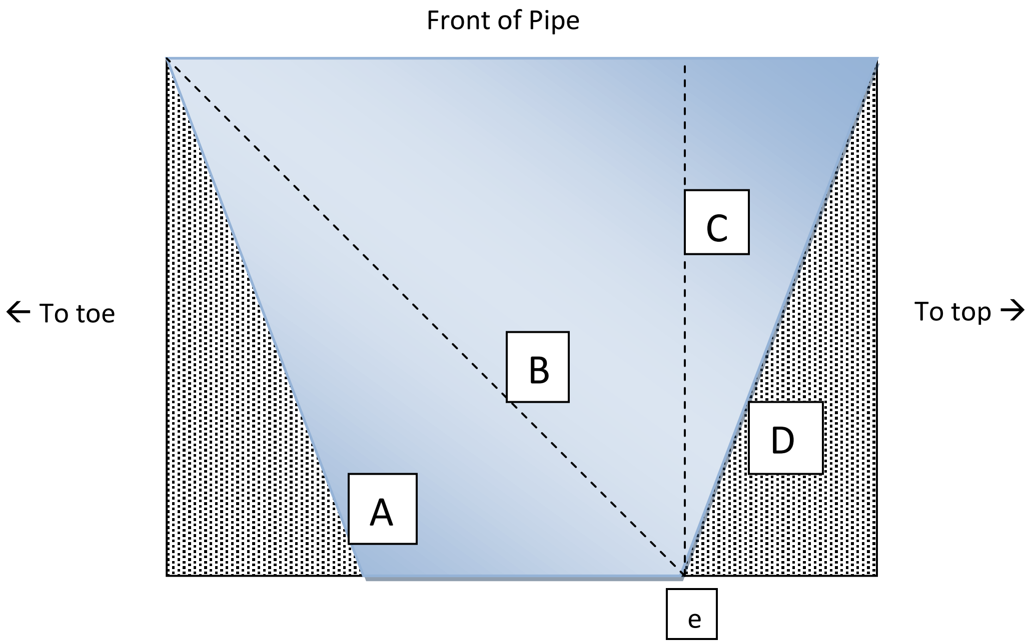
Now, while that distance won't be too tough to measure accurately, other dimensions can be and, as we said, this is a fairly critical measurement. But you can simplify things. We can locate the upper cut mechanically relatively easily.

If we don't really need to do this math, why bring it up? We still need to consider the distance needed for stopper travel, and this measurement tells us how much of the short side of the mitered section will be left after the wedge is cut. Just as in the original article, we must still allow the stopper adequate range for tuning. If you find that the top cut won't leave enough room for the stopper, you'll have to move the miter cuts farther down, and move your target marks accordingly.

If you haven't yet made a template for the 22½° cut, skip down to the Appendix section on the template, then come back.

With the fence on your template up against the back of the pipe, and with the template pointing toward toe pipe's toe, (to the left, in the diagram), lay a straightedge across the pipe up against the template and line it up with the mark you made when you determined the top limit of the lower cut. Mark a line across the pipe at the target point with the straightedge. That's Line A in the diagram below.

Now, with a speed square or tri-square aligned with the point at which that line crosses the front of the pipe, lay out a second line on a 45° angle (Line B). From the point at which line B crosses the back of the pipe (point "e"), use your template to draw a 22½° line (Line D) back to the front of the pipe. Now, check the distance between lines A and D at the front of the pipe. It should match the distance you calculated earlier. (An alternate method: from point "e," extend a 90° line to the front of the pipe (Line C). The distance from where Line C crosses the front of the pipe to the point where lines A and B meet should exactly equal the depth of the pipe.)




The blue shaded area is the wedge, which is a trapezoid (specifically, it is an Isosceles trapezoid, which means it is symmetrical—the side legs are both the same length). The gray shaded areas are there just to aid understanding.

There are several ways to check the accuracy of the layout lines. We'll describe one easy method but you can use trigonometry ratios to get right down to the gnat's eyebrow if you want. There are a number of online trig calculators available, or you can use any calculator that has trig functions on it. The most important point to remember is to "measure twice, cut once" and double check everything (including that the long side of the wedge is on the front of the pipe) before you begin the cut.

The markings on the pipe should end up looking something like this (the blue tape is to enhance visibility):

Figure 2 - Both cut lines are marked and ready to saw

Since the trapezoid is symmetrical, the two outside (gray-shaded) triangles in the diagram are congruent—they have identical shape and size. The two short sides of the gray-shaded triangles (at the bottom) are equal. That will be important later in the Appendix, where we explain the geometry in more depth for those who are interested, but not just now.

To eliminate measurement errors due to irregularities in the surfaces of the pipe (chips, rough edges, etc.), use a tri-square to mark the cut lines an equal distance in from each edge. Because the trapezoidal wedge is symmetrical, the diagonals (Line B and its mate) are of equal length, or congruent. If we shorten each leg by the same amount, the resulting diagonals from the shortened legs will also be congruent. This allows us to measure from these marks to the corresponding mark along the diagonal rather than from the ends, eliminating measurement errors caused by irregularities on the surface of the pipe. If the lengths of the two diagonals match exactly, you have laid out your cut lines perfectly.

You can see that while this is a little more complex, it is not terribly so. Accuracy is still paramount. The same guidelines for surface preparation and mechanical connection of the joints apply. You might find it easier to sand the wedge by anchoring the big sanding block and moving the wedge, rather than the other way around.

Here’s how the process goes after you make your cuts:


Figure 3 - Separate the cut sections…

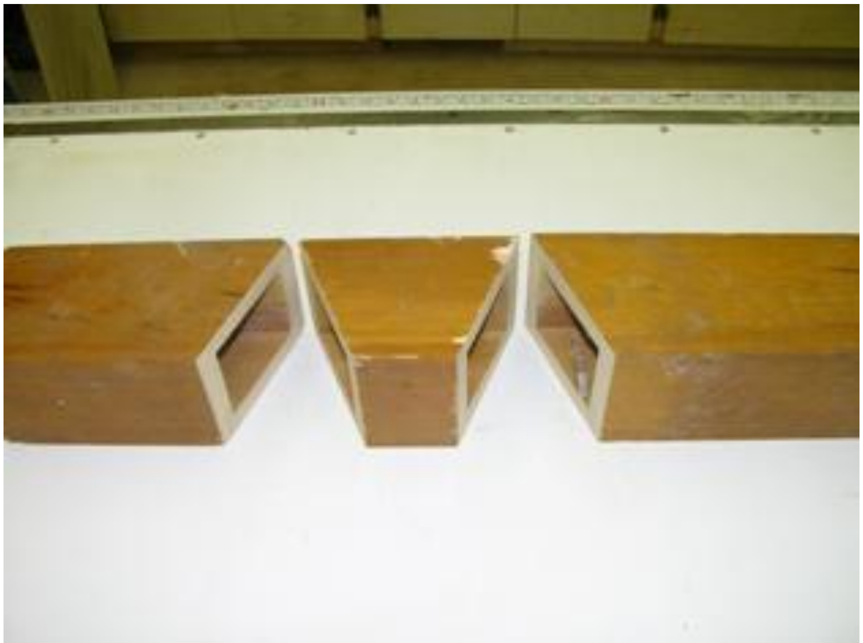


Figure 4 - Flip the wedge over…

Figure 5 - Lay out the pieces in the final shape

Figure 6 - Test the fit, and you're ready to assemble

Glue and clamp the pipe-to-wedge joint first and allow the glue to set up and cure, preferably overnight. Then continue on with the wedge-to-mitered-section joint. This helps to assure that you won’t damage the uncured bond when you clamp the second joint.

Keep in mind that since these joints are closer to being end-grain joints than long-grain, they will not be as strong as a single 45° joint. Additional mechanical strengthening—screws, splines, dowels, or biscuits—is highly recommended.

It isn’t likely you will get a perfectly dimensioned, absolutely flat cut unless you have a very good, sharp handsaw and very good handsaw skills, so you will probably have to do a little more cosmetic work on the joint. But with care you should achieve a result you can be proud of.

Some builders wrap the seams with leather after the pipe is reassembled. This is optional, but serves two purposes: it hides any cosmetic imperfections such as rough edges from the saw, and provides assurance that the pipe doesn’t leak air, which would give you problems. If you choose to wrap it, you’ll need to sand the finish off where the leather will be glued to the joint.

This method will give you a result that keeps the proportions consistent and the transition from vertical to horizontal looking right because the size of the wedge increases as the depth of the pipe increases.

## APPENDIX

### The Math

All of the dimensions you need to verify your cut line layouts can be determined from two simple formulas. Both are based on the depth of the pipe, and since we’re dealing with symmetrical pieces, we’ve simplified things for you. We’ll give you the formulas then explain how they’re derived. If you aren’t interested in the math, just skip the explanation.

Here goes:

“D” is the depth of the pipe.

The length of the long side of the wedge (“L”) is given by the formula:

$$L = D \times \sqrt{2}$$

The short side of the wedge (“S”) is given by the formula:

$$S = D \times 0.586$$

**The Explanation**

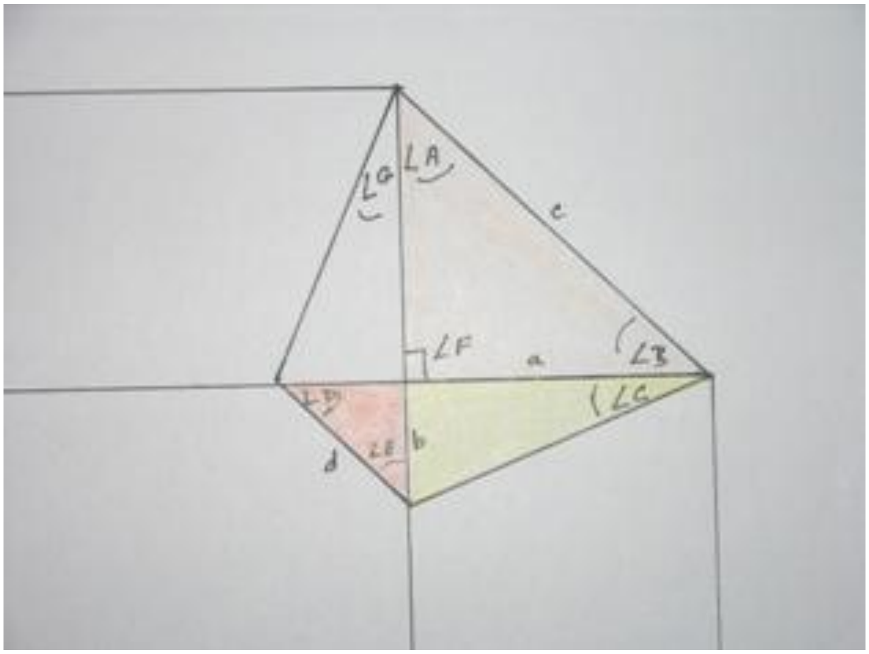


Figure 7 - Diagram of the wedge; the shaded triangles are explained in the text.

To understand how we arrived at the second ratio, refer to the diagram and follow along. We first define four terms (note that the letters we use here are different than in the earlier diagram):

- ***base angle*** is the angle from which your measurements originate. In the large triangle, Angle *A* or Angle *B* could be a base angle;
- ***hypotenuse*** is the side opposite the right triangle. In the large triangle, it would be side *c*.
- ***adjacent*** side is the side connecting the right angle and the base angle. In the yellow triangle, side *a* is the adjacent side, since we're referencing off of Angle *C*.
- ***opposite*** side is the side opposite the base angle, running between the adjacent side and the hypotenuse. In the yellow triangle, side *b* is the opposite side.

In this diagram, Angles A, B, D, and E are all 45°; angles C and G are 22½°. Angle F is 90°, and we'll apply that letter designation to all four of the angles at that intersection.

Side *a* is equal to the depth of the pipe;
Side *b* is the side opposite angle C;
Side *c* is the hypotenuse of the of the right triangle AFB (the large one, lightly shaded);
Side *d* is the hypotenuse of the right triangle DFE (the small one, shaded in red);

When the two sides of a right triangle are equal (if $a = b$) we can restate the Pythagorean Theorem ($a^2 + b^2 = c^2$) like this:

| | | |
|---|---|---|
| $c^2 = 2a^2$ | ; | Combine like terms, since $a = b$ |
| $c = \sqrt{2} \times \sqrt{a^2}$ | ; | Take the square root of each side |
| $c = a\sqrt{2}$ | ; | ($\sqrt{a^2} = a$) |

Trigonometry allows us to go a step beyond Pythagoras. To use the Pythagorean Theorem to determine the length of side *b* we would have to know the length of the hypotenuse of the triangle shaded in yellow. Since we don't know the hypotenuse but we do know the value of angle *C*, we can use the Tangent[1] ratio to find the length of side *b*.

We know the angle and length of the adjacent side, thus we can determine the length of the opposite side *b* by multiplying the Tangent of the base angle *C* by the length of the adjacent side:

$$b = a \times \tan(C)$$

If angle *C* is 22½°:

$$b = a \times \tan(22.5)$$

The tangent of 22½° is 0.41421, so the length of the opposite side (side *d*) will be 0.41421 times the depth of the pipe. In the diagram in the first section, this is the length of the short side of either of the gray-shaded triangles. That side forms one side of the smaller (red-shaded) triangle, and the hypotenuse of that triangle is the length of the short side of the wedge.

Substituting that value back into the earlier restated formula for the hypotenuse (side *d*), we get:

$$d = a \times 0.41421 \times \sqrt{2}$$

Here's the whole sequence for calculating the short base of the wedge:

*a* = depth of the pipe
*b* = length of opposite side in the yellow triangle (or, side of the red-shaded triangle)
*d* = hypotenuse of the small (red-shaded) triangle, which is the short base of the wedge

| | | |
|---|---|---|
| $d = b\sqrt{2}$ | ; | Restated Pythagorean Theorem |
| $d = a \times \tan(22.5) \times \sqrt{2}$ | ; | Express *b* in terms of *a* |
| $d = (0.41421 \times \sqrt{2}) \times a$ | ; | |
| $d = 0.58579a$ | ; | Simplify |

We'll shorten it to 0.586. This ratio will be constant, because all of the angles are constant. The only variable is the depth of the pipe.

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[1] The ratio of the lengths of the opposite side to the adjacent side is the Tangent. That ratio is a constant for any base angle.

Consider the "3-4-5 Right Triangle" from high-school geometry. The ratio of the opposite side ("3") to the adjacent side ("4") is ¾, or 0.75. 0.75 is the Tangent of the angle. We can determine the angle with a reciprocal function on the calculator (or with a trig table, but calculators are easier and faster). The calculator tells us that the angle whose tangent is 0.75 is 36.87°. No matter what length an adjacent side is, if the angle is 36.87°, the opposite side will be ¾ the length of the adjacent side.

To prove the construction steps of the diagram in the instructions, consider what we know:

- If we extend lines perpendicular to the faces of the pipe at the end points of the long base of the trapezoid, we'll have a rectangle with height $a$ and length $L$.
- Since the trapezoid contained within that rectangle is symmetrical, the outside triangles (the gray-shaded ones in the diagram) are congruent. Therefore, the short sides (the opposite side) of either outside triangle are identical, with a length given by:
  $a \times \tan(22.5)$, or $a \times 0.41421$.
- The 45° line going to point "e" describes a 45-45-90 right triangle, meaning the two sides joining the right angle are equal to the depth of the pipe, or $a$.
- Since we're only interested in the length of the short base of the trapezoid, we subtract the length of the opposite side of the shaded triangle, which we calculated using the tangent of the angle, from the length of the side of the 45-45-90 triangle:

  $S = a - 0.41421a$
  $S = 0.58579a$

  The result matches what we calculated earlier. Therefore, we know that our procedure results in the correct proportions.

Remember that the key dimension is L; S is useful primarily to double-check your measurements.

## Making the Template

The accuracy of the cuts will be only as accurate as your template. Solutions to achieve the required accuracy range from the high-tech digital protractors and angle finders and "medium-tech" adjustable woodworkers triangles that are available from woodworking specialty shops and some home centers, to the low-tech compass and straightedge. Properly used, any of the solutions will give you accurate results.

Square up a board that's at least half as wide as your deepest pipe. If you have a power miter saw that will get through a board of that width, set it to 22½° (it probably has a stop at 22½) and make the cut.

If you have a digital protractor or a table saw with a dead-accurate miter gauge, set it to 67½° (we want the 22½° cut to be relative to the width of the board, not the length), square it up to the board, and mark the line.

If you aren't certain how accurate your miter gauge is, we can use a simple technique you probably learned in high school geometry class to lay out a very accurate line (and you thought you'd never use that knowledge in real life!). You'll need a speed square or tri-square, a compass, and a straightedge. Follow the steps below in the section "Bisecting an Angle" to lay out your cut line.

If you use this method to lay out the cut, loosen the miter gauge and adjust it so the 22½° line on the template is perfectly aligned with the miter slots on the table saw and secure it. If you're using a circular saw, clamp a fence to the board to assure a straight cut along the line.
Make a test cut some distance away from the actual line and measure to confirm the line is exactly parallel to the cut edge. If you find it isn't perfectly parallel, adjust your miter gauge or fence and shave off just a bit more. Repeat until the setting is perfect.

Here's what our template looks like in use. Five minutes, two pieces of scrap wood, and two screws was all it took. (This was taken after we marked the second (upper) cut line.)

Figure 8 - A simple template for laying out 22½° lines

We attached a board to the edge of the template to act as a fence. This will ensure that damage to the corner of the pipe doesn't become an issue that affects the accuracy of our layout. If the template isn't stable when it's up against the pipe, you know you have some other issues to deal with before you make the cut. A degree or two of error will become very apparent when you reassemble. Having the board extend both above and below will allow you to flip the template over, which you'll need to do.

## Bisecting an Angle

Mark a line across the board at 90°. Using a speed square or tri-square, draw another line at 45° to the first, with the two lines intersecting a bit inside the edge of the board (that will make bisecting the angle easier). We'll use the compass to bisect that angle (to achieve our desired 22½°) and the straightedge to extend the line all the way across the board.

Although we're illustrating the process beginning with a 90° angle just to make it a little easier to see, the steps are the same for any angle.

- Step one: With the point of the compass at the origin (the point at which the two lines describing the angle meet), make a mark with the pencil leg on each line.

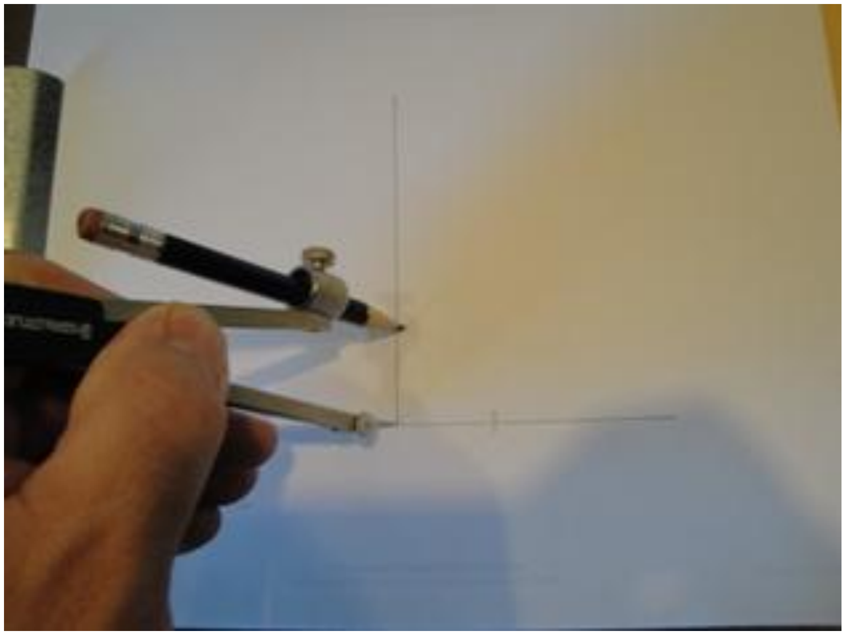

Figure 9 - Mark both rays of the angle

- Step two: without changing the setting of the compass, move the point of the compass to the mark you just made on one leg and draw an arc that crosses the approximate midpoint of the angle you are bisecting. Repeat for the other leg.

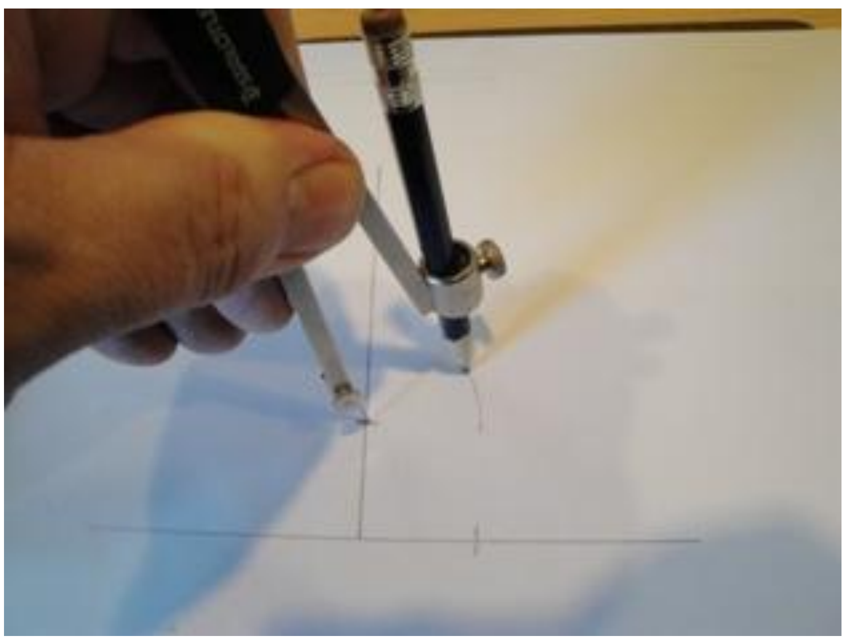

Figure 10 - Draw an arc from the mark to the inside of the angle

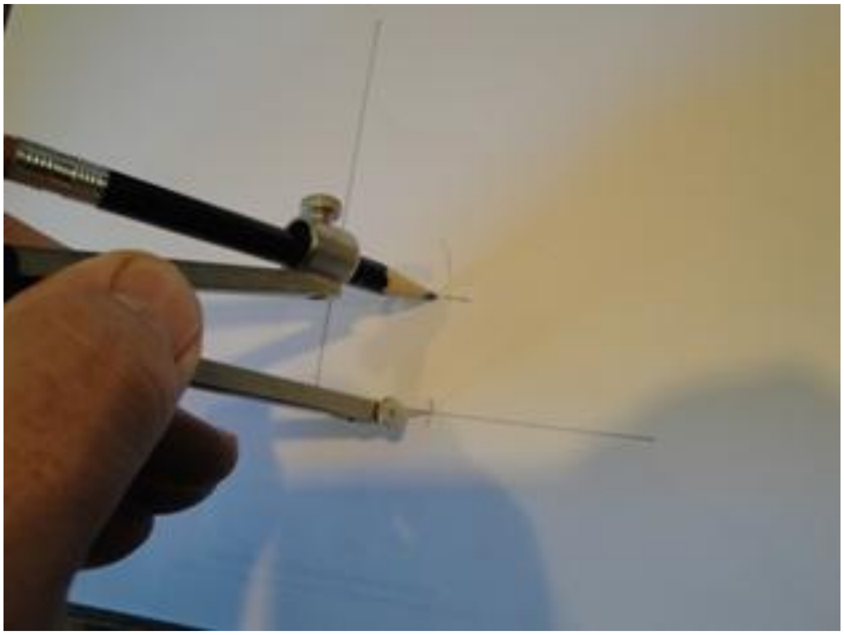

Figure 11 - Repeat for the other ray

- Step three: Using the straightedge, draw a line from the origin through the point where the two arcs intersect. You have now bisected the 90° angle. The ray you have just drawn is 45°. If you repeat the process with this angle, you will create a 22½° angle.

Figure 12 - Draw a line from the origin through the intersection of the two arcs

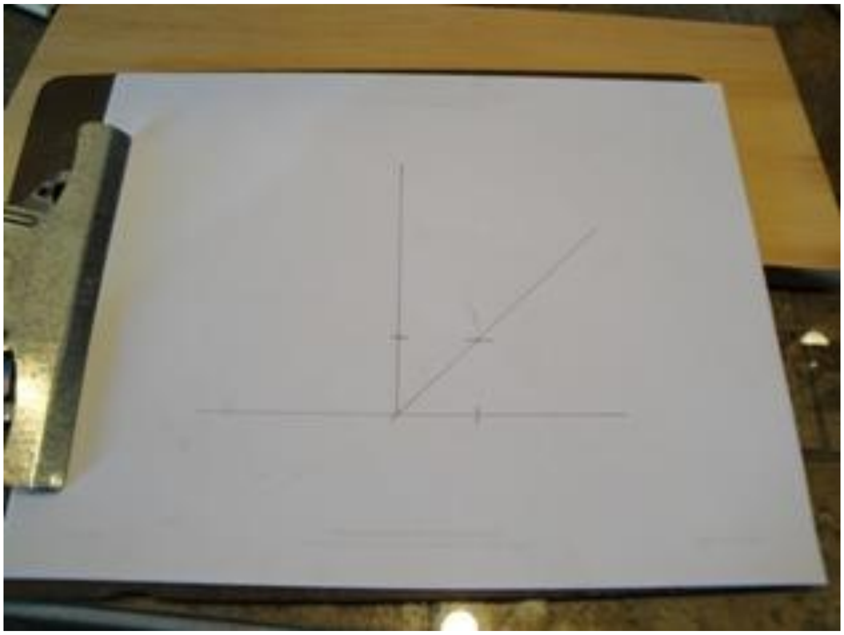

Figure 13 - You now have a 45° angle. Follow the same steps again, using the 45° ray, and you can create a 22½° angle